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THE 11-YEAR SUN-SPOT PERIOD, SECULAR PERIODS OF SOLAR ACTIVITY, AND SYNCHRONOUS VARIATIONS IN TERRESTRIAL PHENOMENA

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[Arcade, N. Y., April 1933]

SYNOPSIS

This paper supplements a former one with corrections and additional matter. A few changes are made in the Fritz epochs of "probable maxima" of sun spots, dating from 300 A.D., and it is shown that the frequency distribution of the 11-year sun-spot intervals derived from the ancient epochs has about the same mean, skewness, and dispersion as that of the Wolfer intervals from 1610. For the whole period of 1,600 years the most frequent interval or mode is computed to be 10.94 years while the normal length of the period computed by a least-square method is 11.067 years. The mean deviation from 11.0 years is ± 1.69 years.

By appropriate statistical processes and criteria, the sequence of the 11-year intervals is shown to be systematic rather than fortuitous. While the most frequent interval between peaks or hollows in a random sequence is the two-interval, there is a marked tendency for maxima or minima in the solar curve to recur about every third interval. In other words the most frequent interval of recurrence is about 36 years.

The epochs of maximum and minimum length of the 11-year period, derived from the curve of 11-year intervals, yield by the least-square computation a normal length of 37.5 years for the long period, with an amplitude of 2.4 years. On eliminating the 37-year period by an appropriate smoothing of the 11-year intervals, a still longer period is disclosed with a normal length of about 83 years and an amplitude of 1.5 years. Further smoothing discloses a 300-year period with an amplitude of 0.5 year. The 300-year period undergoes a long secular variation in length, roughly estimated at 1,400 years.

Both the 37-year and the 83-year periods undergo a 300-year variation in length, comparable with that of the 11-year period, the maximum lengths being about twice the minimum lengths.

These three periods exist also in the relative numbers and the ratios, $a:b$, that is, time of increase to time of decrease of sun spots from minimum to minimum, the numbers varying inversely and the ratios directly with the length of the 11-year period.

These periods are apparent not only in auroral data but in various other terrestrial data—frequency of severe winters, frequency of Chinese earthquakes, flood and low stages of the Nile, tree growth in Arizona and California, and wheat prices in England.

The epochs of maxima of the three periods lag somewhat behind the epochs of maximum solar activity, and the amount of the lag is proportional to the length of the period. The lags of the 37-year and 83-year epochs

exhibit a 300-year period, also a long secular variation—the lag after 1,000 A.D. being about two thirds that previously.

INTRODUCTION

In a former paper (1) I discussed the so-called "Brückner meteorological cycle" of 35 years and showed that a similar variation could also be traced in certain solar data. This indicated that the Brückner climatic cycle probably is of solar origin. A 300-year period was also shown to exist in the data. The present paper supplements the one just mentioned, corrects an error in the ancient epochs, gives additional evidence for the 35-year and 300-year periods and new evidence for the existence of two periods of around 83 years and 1,400 years in both solar and terrestrial data. It deals mainly with new results and the reader should refer to the former paper in order to have a clear comprehension of some of the details. Others of my papers contain material which will be referred to either as results previously obtained or as discussions of methods employed in the present paper.

As to the truth of the conclusions offered, it may be stated that, while the individual steps leading to the final results have varying degrees of validity due to the inherent inexactness of the data, by the smoothing processes and graphical methods employed inaccuracies can largely be detected and eliminated. The validity of the whole body of evidence rests upon the mutual consistency of its separate elements.

THE 11-YEAR SUN-SPOT PERIOD

The unbroken continuity of the 11-year sun-spot period is a fundamental concept in this investigation. If it can be shown that this period has existed continuously, although with variations in length, for an indefinite duration, the reasonable inference is that other solar periods may be equally continuous and persistent over long intervals.

My early paper contained a discussion of the ancient epochs of sun-spot maxima derived by Fritz from early auroral data and the Chinese observations of sunspots. His paper was translated and published in the MONTHLY WEATHER REVIEW, October 1928 and readers should refer to it in connection with the following discussion.

A careful examination of Fritz's tables—1, Sun-spot epochs; 2, Auroral epochs; 5, Probable maxima—has led me to make a few changes in his epochs of maxima. The Fritz, Lovering, and Short catalogs of auroras were examined for possible additions or discrepancies. From

the Lovering catalog were obtained four additional years of auroral displays, near the probable dates of epochs missing in the Fritz list, viz, 629, 752, 1039, 1499.

There are at least two cases where the solar data seem doubtful. During the years 535-536 the sun had diminished brilliancy for 14 months, and in 626 the sun was partially darkened for 8 months. Fritz regarded 538 as a probable maximum, based on 535 as a solar epoch and 540 from auroral data. I reject the year 535 and regard 540 as the probable epoch. He regarded 625 as a maximum, based on 626 as a solar and 624 as an auroral epoch. I reject 626 and advance the date of the epoch to 629, the date of an aurora in Lovering's catalog. The year 657 is regarded as a maximum epoch by Fritz, being a mean of

table 5. In table 1, 1547 is a sun-spot year, while 1549 is given as the maximum. Assuming 1547 to be the correct date, the probable maximum in table 5 is more likely 1548.

After 388 only four epochs are based solely on sun-spot observations. I have changed a few epochs, giving greater weight to two or more adjacent years with aurora than to a single year with sun spots. Such epochs are 870, 970, 1204, and 1604 instead of 872, 972, 1203, and 1603. The epoch 360 has been changed to 361 to make the interval from the preceding epoch 7 years instead of 6, which is an improbably small interval.

The epochs remaining for which no data exist have been inserted at nearly equidistant intervals between the

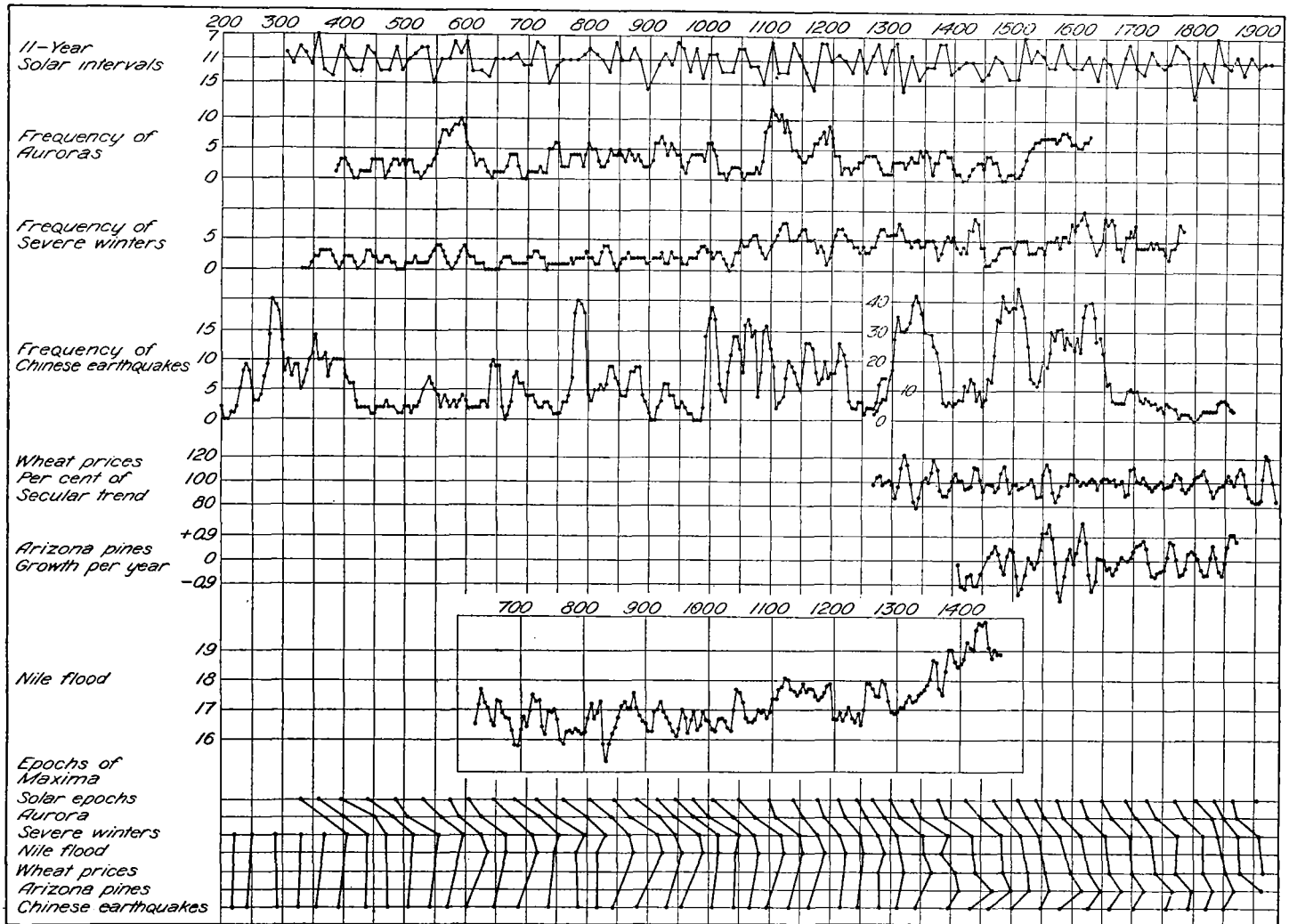


FIGURE 1.—The 37-year period in various solar and terrestrial data. Epochs of maxima are plotted below and joined to show interrelations and lags.

two aurora years, 654 and 660. However the year 660 is not given in either of the two large catalogs and I have therefore regarded 654 as the more probable date.

As a result of these changes, instead of one epoch between 512 and 538 and one between 538 and 555, I insert two between 512 and 540 and none between 540 and 555, and instead of three epochs between 616 and 657 I assign only two epochs between 616 and 654.

Some discrepancies and errors have been found in Fritz's tables. He omitted a few epochs from table 5 which seem clearly evident from table 2, namely, 479, 488, and 879. The epoch 1280 appears in table 5 but not in table 2, while 388 appears in table 1 but not in

epochs derived from observational data. The finally adopted epochs, with the supplied epochs indicated by an asterisk, are given in table 1, together with the Wolfer epochs of maxima. The 11-year intervals are given in column 2 and are shown plotted in figure 1 on their mid-dates.

Evidence as to the accuracy of these epochs is afforded by their recurrence in nearly the same sequence after an interval of 1,184 years. This interval is 107(11.066), 32(37.0), 14(84.5), 4(296). It is therefore a nearly exact multiple of three periods, which will be discussed below, and is approximately a long secular variation. Adding this interval to the epochs in table 1, beginning with 301

and ending with 742, there results a series of dates which coincide closely with the observed epochs. The mean deviation of the computed from the observed epochs is ± 1.78 years, and 90 percent of the deviations are within the limits ± 3 years. Now the average deviation from 11.1 years of the intervals between maxima from 1615 to 1928 is ± 1.61 years and 85 percent are between 8.1 and 14.1 years. *Thus a prediction of a maximum epoch made by adding 1,184 years to the somewhat uncertain early epochs is nearly as accurate as one made by adding 11.1 years to the relatively exact epochs since 1615. Such a result confirms in a striking manner the general accuracy of the ancient epochs and the reality of the various solar periods.*

Frequency distribution of the 11-year intervals.—The frequency curve of the 11-year intervals shows a slight positive skewness. Omitting the long 17-year interval, the 55 Wolfer intervals yield a mean of 11.07, and a mode 10.94; while for the Fritz revised intervals the mean is 11.04 and the mode 10.94. The mean deviation of the Wolfer intervals from 11.1 years is ± 1.61 for the maxima. For the Fritz intervals, the deviation from 11.0 is ± 1.70 . The mean variability of the Wolfer intervals is ± 2.71 ; of the Fritz intervals ± 2.46 .

The normal period length and normal epochs.—In a former paper (3), I called attention to Newcomb's (6) method of evaluating the normal epochs and normal length of period. He derived by a least-square solution the normal length of the period from 1610 to 1900 as 11.13 years. A variation of his method is represented by the equation

$$b = \frac{6}{n^3 - n} \left[(n-1)(y_n - y_1) + (n-3)(y_{n-1} - y_2) + \dots \right]$$

in which y_1, y_2, \dots, y_n are the epochs of maxima, n the number of epochs, and b the normal length of the period.

The computation of the normal values from the combined Fritz and Wolfer epochs is facilitated by averaging the epochs in groups of seven, which gives a series of 21 mean epochs beginning with 332 and ending with 1882. Denoting the differences between the y 's as c 's and the coefficients as w 's, the work is further shortened by employing $c-77w$ instead of c . The formula then

becomes $7b = 77 + \frac{6\sum [w(c-77w)]}{n^3 - n}$. Computing, $b = 11.067$.

The normal mid-epoch is 1106.13. The residuals of the observed from the normal epochs are given in table 1.

Methods of statistical analysis and application to solar data.—One of my former papers (2) contained a discussion of the statistical criteria for the detection of a systematic order of succession in any series of data. Obviously, if the sequence of any given data is indistinguishable from that shown by a series of random numbers, no periodicity can be present. It was there shown that the sequence of the deviations in the length of the sun-spot period from a normal period, instead of being accidental, as Newcomb concluded, is systematic to a marked degree.

Analysis of the intervals from the combined Fritz and Wolfer epochs of maxima, table 1, necessitated the averaging of three drawings from a bowl of the 147 values to determine approximately the normal random distribution of the intervals between peaks or hollows. This distribution differs from that for an infinite number of random values as determined by Besson (7), owing to the small number of different values, 11. The average interval for the random series (drawings) is 3.18, for the natural data 3.48; the modal or most frequent interval is about 2.40 for the random series, but about 3.25 for the natural data. The essentially systematic character of the natural data

is well shown by this statistical analysis. Since the unit interval is 11.1 years, the most frequent interval in years is 3.25×11.1 , or 36 years.

Where such persistent deviations from a random sequence occurs, it can only be regarded as due to the existence of a definite periodicity of variable length, whose average length should theoretically differ little from the value of the mode above derived, 36 years. The reader is referred to my papers (4) and (5) for a discussion of variable periods and the methods of investigation employed in the present paper.

THE 37-YEAR PERIOD

It has been shown that in the sequence of the 11-year intervals there is a recurrence of extreme long or short values about every 36 years. With this provisional length of the period we draw the smooth curve through these intervals (fig. 1). While the most frequent interval between these extremes is about 36 years it is seen that there are quite wide deviations from this value in the actual intervals. These deviations are due partly to inaccuracies in the data but mostly to a systematic variation in the length of this interval, which will be discussed below.

The 37-year epochs.—The epochs derived from the smooth curve are given in table 2. There is some uncertainty regarding the epochs since 1850. They are designated as epochs of the 37-year period. The epochs which date the short 11-year intervals are called epochs of maxima, and vice versa. The normal length of the period derived by the least-square method from 300 to 1900 is 37.5 years.

Referring to my 1905 paper (1) it will be noted that the 37-year epochs given there agree closely with my recent determinations except in the sixth century, where, owing to the changes made in the Fritz epochs, above noted, one maximum epoch and one minimum epoch have been omitted.

Other solar data showing a 37-year period are the relative numbers at maxima and the ratio a/b , (a , the ascending; b , the descending branch of 11-year curve) as shown in chart 1 of my 1905 paper. *The relative numbers vary inversely with the length of the 11-year period with an average lag of 5 years, while the ratios vary directly with an average lag of 7 years.*

Amplitude of the 37-year period.—The amplitude of the period is derived by averaging the 11-year intervals at the epochs of maxima and minima of the period. For the period from 1610 to 1920 the averages are, long 13.8 years, short 9.2 years; whence the amplitude is 2.3 years. Between 300 and 1600 the 11-year intervals from maxima only are available, and obviously the range derived from the maxima and minima of the curve will be less than the true range. The averages are, long 13.1 years; short, 8.9; whence the amplitude is 2.1 years.

THE 83-YEAR PERIOD

The 83-year variation in the length of the 11-year period.—In order to disclose periods longer than 37 years in the series of 11-year intervals, it is necessary to employ an appropriate smoothing formula. The 11-year intervals smoothed by the formula $(a+2b+2c+d) \div 6$ are plotted in figure 2, curve 1. The epochs of maxima and minima of the curve are given in table 3. The normal length of the period by the least-square method is 83.1 years and the normal mid-epoch is 1132.4.

Amplitude of the 83-year period.—The amplitude of the period can be approximately derived by averaging the

maxima and the minima of the curve and applying the proper factor to correct for the reduction in the range effected by the smoothing formula. The mean amplitude after applying this reduction factor is 1.52 years.

The 83-year variation in the relative numbers and in the ratio a/b .—The relative numbers at spot maxima and the ratios smoothed by running means of seven terms are shown in figure 2. Comparing these curves with curve 1 the following relations for the 83-year variation in various solar data are derived. *The relative numbers vary inversely with the length of the 11-year period with a lag of about 10 years, while the ratios vary directly with a lag of*

The amplitude of the 300-year variation.—A smoothing of the 11-year intervals by successive summations of 11 terms further smoothed by summations of 7 terms satisfactorily eliminates all periods shorter than 300 years. The amplitude of the period can be approximately derived by averaging the five maxima and five minima of the curve and applying the proper factor to correct for the reduction in the range effected by the smoothing. The result is 0.48 year, or about one-third that of the 83-year period and one fifth that of the 37-year period.

The 300-year variation in the residuals.—The residuals in table 1, averaged by half-century intervals, are plotted

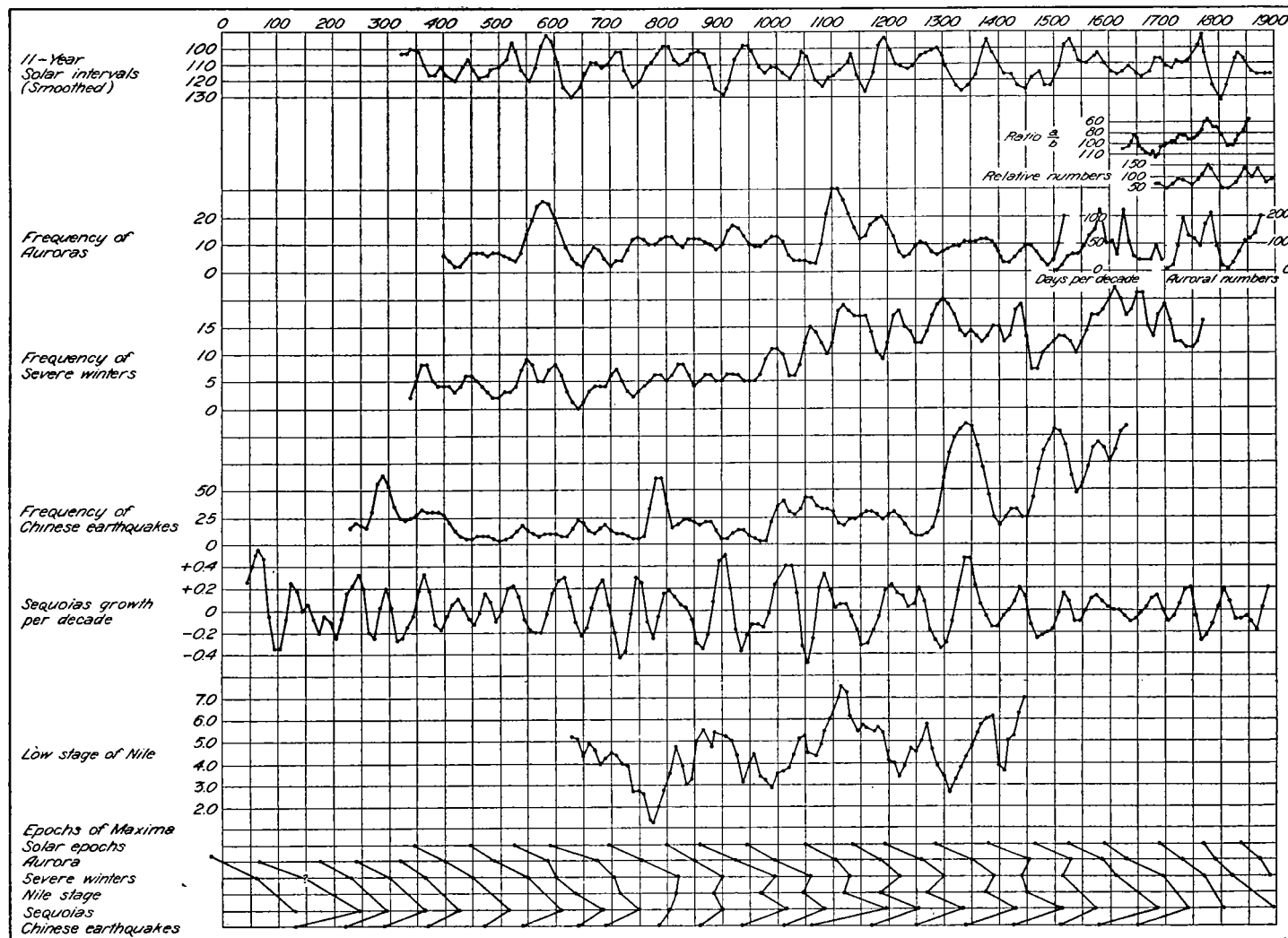


FIGURE 2.—The 83-year period in various solar and terrestrial data. Epochs of maxima are plotted below and joined to show interrelations and lags.

about 15 years. These relations are the same as with the 37-year period, but the lag is about twice as great.

THE 300-YEAR PERIOD

The 300-year variation in the length of the 11-year period.—The averages of the 11-year intervals for each half-century are shown plotted in figure 3, and the epochs of maxima and minima are given in table 4. The average length of the period is about 300 years but varies between 225 years around 400 A.D. and 1650, and 375 years around 1100 A.D. This variation in the length of the period indicates a still longer secular variation which may be roughly estimated as around 1,400 years.

in figure 3, and the epochs derived from this curve are given in table 4. The epochs of maximum minus residuals are called epochs of maxima, while the epochs of maximum plus residuals are called epochs of minima.

The 300-year variation in the solar activity and in the ratio a/b .—As stated in my 1905 paper (1) (p. 66), and shown in figure 2 herewith, a minimum of solar activity prevailed about 1680, associated with a maximum value of the ratio a/b . The reverse conditions prevailed around 1780. Thus with the long period as with the shorter periods the solar activity varies inversely with the length of the 11-year period.

The 300-year variation in the length of the 37-year period.—The intervals between like phases of the 37-

year epochs in table 2 were smoothed by successive means of three terms. The resulting values are shown

of the 37-year period, given in table 4. These epochs show a 300-year variation in the length, which ranges between 27 and 50 years.

The 300-year variation in the amplitude of the 37-year period.—The range of the 37-year period is the difference in years between adjacent peaks and hollows of curve 1, figure 1. These ranges, smoothed by five term averages, show a 300-year period with the epochs given in table 4. Owing to the crudeness of the original data, the agreement between the two series of epochs is not very close, but the averages of the nine epochs in each series agree within 8 years, showing that the length and amplitude of the 37-year period vary directly with each other.

This is an extremely significant and important result and the relation may be expressed in very general terms as follows: *If a series of data shows a systematically varying period, the amplitude varies directly with the length of the period.* This seems to be a law of universal application for all periods which vary in length, both solar and meteorological.

The 300-year variation in the length of the 83-year period.—The intervals between the 83-year epochs of maxima and minima are shown plotted in figure 3, curve 2. These range between 55 and 110 years. The epochs of the 300-year period derived from this curve are given in table 4. These epochs have long intervals, averaging 330 years, around 1250 and short intervals, 290 years, in the early and latter part of the series, indicating a long secular variation in the length of the 300-year period.

It will be seen that curves 1 and 2 have nearly reversed phases. The persistence of this feature for 1,500 years is highly significant and is strong evidence for the reality of the two periods.

PERIODICITY TABULATIONS

Tabulation of the 11-year epochs.—If the positions of the 11-year epochs of maxima or minima are plotted in a table with rows 11 years apart and the epochs joined, forming a zigzag line as in figure 4, the variations in the period length are graphically indicated. Every fifth row gives the date of the zero column. The zigzag line is essentially a plot of the accumulated sums of the departures of the 11-year intervals from 11 years. It is also a plot of the residuals, table 1, the zero line being the straight line of best fit, shown as a heavy full line.

The most conspicuous variation is the 300-year period shown by the dotted curve. The integration or successive summation of departures exaggerates the amplitudes of the long periods. The long interval around 375 years midway in the curve and the short intervals around 225 years in the early and latter portions are clearly evident. Upon the long variation are superposed the shorter 83-year and 37-year variations. The latter are indicated approximately by circles at the right, and the former by circles at the left. The opposite phases are midway between the phases thus indicated. Theoretically the phases of a mass curve are advanced one fourth the wave length and in this case the epochs of the residuals, table 4, are shifted about 75 years from the epochs of the length of the period.

Tabulation of the 11-year intervals.—These tabulations contain more than one maximum or minimum phase of a period to the row. To show the 37-year period, the intervals are tabulated as in figure 5 in rows of 27 intervals each, or approximately 300 years to the row, and the maxima averaging 37 years apart are underscored. Each maximum is joined to the eighth maximum, preceding and following. The date of the beginning of the interval in the zero column is at the left. Since the mean length of the rows is 300 years, or nearly four times the 83-year

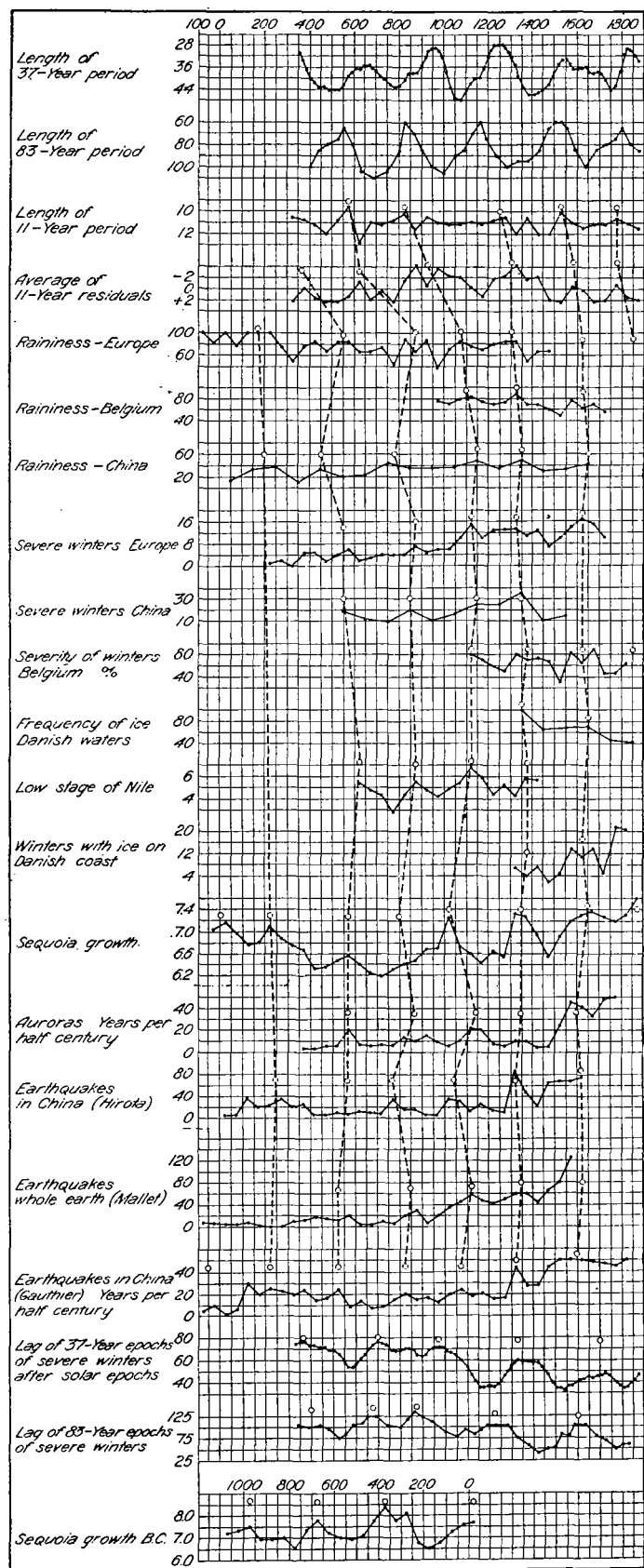


FIGURE 3.—The 300-year period in various solar and terrestrial data. Maxima are indicated by open circles and joined by broken lines.

in figure 3, curve 1. From this curve are derived by simple inspection the epochs of maximum and minimum length

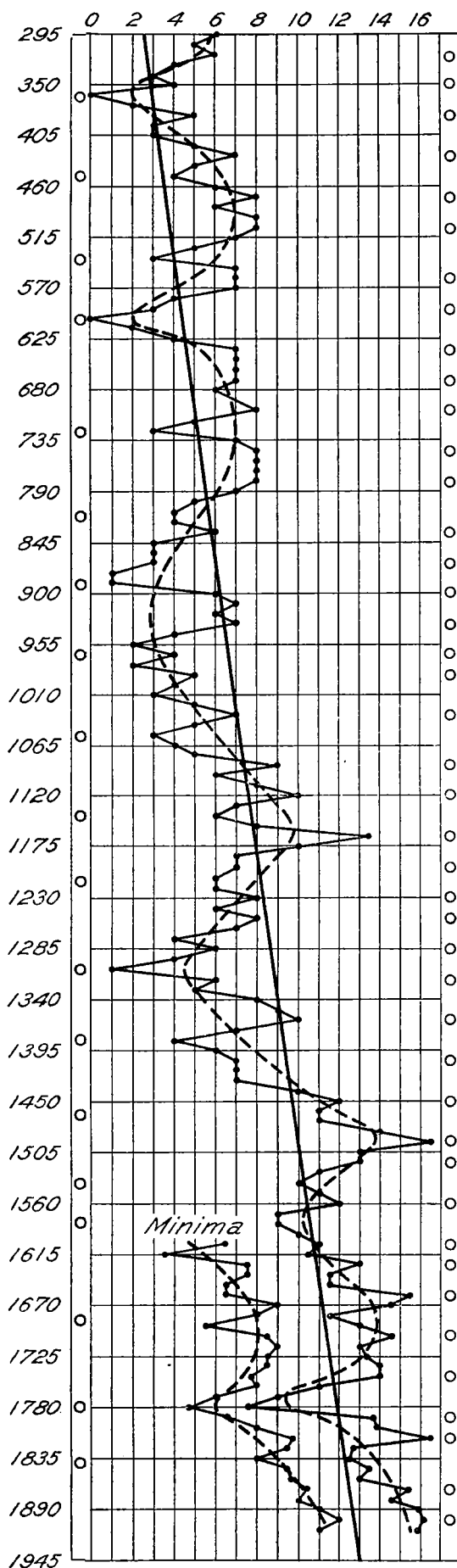


FIGURE 4.—Epochs of sun-spot maxima. Date of zero column for every fifth row at left. The dots representing the epochs are joined by full lines. The broken curved line shows the 300-year period. The straight line is the line of best fit to the plotted data. Open circles at the right and left indicate extreme variations due to the 37-year and 83-year periods, respectively. Wofler epochs of minima are plotted at left.

period, these two periods are practically eliminated and the general trend of the connecting lines is nearly vertical, indicating a mean length of about 37.5 years.

When the first row is shifted one space to the right, the sum of the first five rows shows the period very clearly. Maxima and minima are indicated by full and dashed underscoring. Similar phases of the 37-year period are spaced about three columns apart near column 4 and about four columns apart near column 20. This obviously is the 300-year variation in the length of the period.

To show the 83-year period, the summations of the 11-year intervals plotted in figure 2, averaging 66.4 years,

Year	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
301	10	12	9	10	12	7	13	14	9	11	13	13	9	10	13	13	9	13	11	10	9	15	11	11	8	10	8	13			
595	8	13	13	14	11	11	11	10	12	12	8	9	15	12	11	11	10	9	10	11	13	8	11	11	9	11	16	12			
690	10	12	10	12	8	9	13	9	14	10	13	13	9	9	12	15	8	13	13	8	10	13	16	8	8	11	10				
1193	11	10	11	13	9	13	10	8	13	9	8	16	10	14	12	12	8	13	12	11	11	14	13	10	11	14	14	7			
1497	14	7	11	9	10	12	12	8	11	12	10	14	10	11	15	10	8	12	13	9	11	12	11	8	9	10	17	11			
1788	17	11	14	7	11	12	10	13	10	12	11	11																			
* Sums	58	52	57	57	48	57	55	48	64	52	43	61	65	54	53	63	54	50	55	59	54	52	53	63	56	48	51	68	48		

* Shifting first row one space to the right and omitting sixth row

FIGURE 5.—11-year intervals in a tabulation with 27 columns and rows averaging 300 years. Dates of intervals in zero column at left. Maxima averaging 37 years apart are underscored and joined to the eighth maximum preceding and following. Sums of first 5 rows (first row shifted 1 space to right) show the 37-year period. The 300-year period is shown by the varying distances between curves.

are tabulated in figure 6 with 30 columns, or 332 years, four times the period length, to the row. The general trend of the lines joining every fourth maximum or minimum, indicated by full and dashed underscoring, is nearly vertical, indicating a mean period length of 83 years. Their curvatures, however, indicate a long secular variation of around 1,400 years in the length of the period. The 300-year period is shown by the short intervals between like phases, averaging six columns or 66 years, centered around column 15, and the long intervals, averaging nine columns or 100 years, around column 0.

Year	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	0	1	2	
323	62	62	60	61	66	70	70	67	72	67	65	71	72	68	67	64	68	61	68	73	67	59	61	57	65	74	79	74	69	65	65			
654	69	65	65	67	66	61	61	65	72	67	65	62	59	59	64	66	64	62	61	62	67	75	77	72	64	59	61	67	69	67	67			
979	69	67	67	69	71	66	61	65	72	72	71	70	68	65	62	70	76	69	59	61	66	67	65	67	65	62	61	62	65	67	74	76		
1308	67	74	76	74	70	60	57	62	69	70	70	74	75	71	69	74	74	67	59	57	61	65	66	63	62	66	69	70	69	66	71	70	61	
1643	70	71	69	64	64	67	63	65	66	65	59	55	62	74	82	74	64	62	64	69	70	72	70	65										

FIGURE 6.—11-year intervals summed by $(a+2b+2c+d)$ in a tabulation with 30 columns and rows averaging 332 years. Maxima and minima of 83-year period are indicated by underscoring. The curves joining every fourth epoch illustrate the 300-year and 1400-year variations in the length of the 83-year period.

This graphical device is a variation of that presented by the writer in a former paper (4) to illustrate the variations in the length of the 28-month solar period.

THE PERIODICITIES OF THE AURORA

The 11-year period.—The parallelism between the variations in the frequency of sun spots and auroras is very close. According to Fritz the average lag of auroral maxima after sun spot maxima is about a year.

The 37-year period.—The list given by Fritz has received some additions derived from the Lovering and Short catalogs. The total number of auroras in the 20-year interval centered on every fifth year is plotted in

figure 1. Most of the maxima are obvious, but in some cases the unsmoothed or original data must be considered in the determination of a maximum epoch and in other cases data are lacking so that interpolation is necessary. The greater amplitude of the 83-year period is the cause of some uncertainty. The epochs of maxima and minima are given in table 2 with interpolated or doubtful epochs indicated by asterisks.

The 83-year period.—The amplitude of this period is greater than that of the 37-year period, and it can therefore be traced back to 400 A.D. with considerable accuracy. To eliminate the 37-year period from the 20-year summations, a summation of the number for a given date and that of the second preceding and following is made. These summations for every tenth year are plotted in figure 2. The marked increase in the numbers from 1525 is due in part to the secular variation with a maximum about 1550 and in part to the increase in available records following the era of the introduction of printing. In selecting the 83-year epochs in table 3, this secular variation was taken into consideration.

The total number of days per decade with aurora from 1500 to 1740, and the Fritz auroral numbers, 1700 to 1870, averaged by decades, are shown plotted in figure 2. From these curves the epochs were derived after 1600.

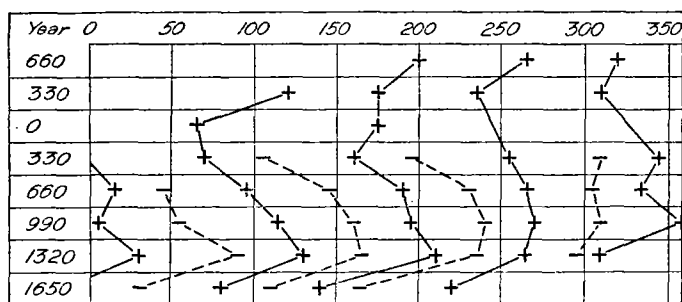


FIGURE 7.—Plus-and-minus signs are 83-year epochs of maximum and minimum auroral frequency. The curvature of the lines joining every fourth epoch indicates a secular variation of around 1400 years in the length of the period.

The occurrence of aurora has been recorded as far back as 503 B.C., and approximate epochs of maximum frequency have been derived from the lists given in the various catalogs. These are given in table 3 with two interpolated epochs indicated by asterisks.

The 300-year period.—The number of years in each half-century from 350 A.D. to 1750, in which aurora was recorded, is plotted in figure 3. The epochs of maximum and minimum frequency are given in table 4.

Tabulation of the 83-year auroral epochs.—Figure 7 shows the 83-year epochs in a table with rows 330 years long. Full lines join epochs of maxima separated by four 83-year intervals. Epochs of minima since 400 A.D. are plotted and joined by dashed lines. The 300-year variation is eliminated from the trend of the lines and their curvature indicates the long secular variation around 1,400 years which has already been noted.

The trend of the lines is on the whole slightly to the left, indicating a period length of approximately 82 years.

The two graphs, figures 6 and 7, are derived wholly independently of each other but the curvatures of the lines are virtually identical.

SEVERE WINTERS IN EUROPE

Records of unusual meteorological events are abundant in European literature. The occurrence of severe winters

has been very consistently recorded, and Brückner, by means of this material, was enabled to extend his 35-year cycle, deduced from modern instrumental records, back to the year 1000. The reader is referred to my 1905 paper (1) for a discussion of his results together with additional results derived from my own researches. Brückner used Pilgram's catalog and began with the year 800 but he regarded the records previous to 1000 as of little value.

To extend the series backward, I have used Hennig's catalog which is very complete. Easton's list was also consulted. Employing the method used by Brückner, the total number of severe winters in the 20-year interval centered on every fifth year were counted and the numbers from 340 to 1030 together with his numbers from 1030 to 1775 are shown graphically in figure 1.

The 37-year period.—Maxima and minima are quite definitely apparent except in a few instances where data are lacking. The maximum and minimum epochs are given in table 2 with interpolated epochs indicated by asterisks.

The 83-year period.—In order to eliminate the 37-year period a smoothing process similar to that used on the auroral numbers was employed, and the smoothed values for every tenth year plotted in figure 2. The derived epochs of maxima and minima are given in table 3. Epochs previous to 300 A.D. are only approximate.

The 300-year period.—In Brooks' "Evolution of Climate" the number of severe winters in Europe per half-century are given from 800 A.D. and I have extended the data back to 300 A.D. from Hennig's catalog. These numbers are shown in figure 3.

FLOOD AND LOW STAGES OF THE NILE

A remarkable series of yearly records of high and low levels of the Nile at the Roda gage, Cairo, from 622 A.D. to 1470 has been published by Prince Omar Toussoun. The original records are in cubits and dated in Mohammedan years. One list from 640 to 1451 was published in 1923. Another list from 622 to 1470 in metric equivalents and corrected to the modern calendar was published in 1925. These two lists differ slightly and after careful examination of the graphs of both lists it was decided to use the first one, making a few corrections to readings, evidently misprints, by comparison with the later list and supplying a number of missing years. Five-year means have been computed for both flood and low stages.

The 11-year period.—The influence of the 11-year solar period on the flood stages is shown by an excessive predominance in the 5-year means of the two-interval over the normal frequency for random numbers, 50 percent vs. 40 percent. As for the minima there is a relative excess of the four- and five-intervals, indicating a 20- to 25-year period.

The 37-year period.—When the pentad means of the flood stages are smoothed by the formula, $(a+b) \div 2$, the 11-year period is eliminated and the longer periods can be recognized. The smoothed means are shown in figure 1. Epochs of the 37-year maxima and minima corrected to the Gregorian calendar are given in table 2.

The 83-year period.—The contrast between the flood and low stages, both in their origin and in the short periods shown by the pentad means, is further shown by the longer periods. The 37-year period is best shown by the flood levels while the longer periods are best

shown by the low stages. Figure 2 shows 10-year means of the low stages smoothed by $(a+b) \div 2$.

The 37-year period somewhat obscures the longer period in the curve of flood stages, but the epochs of the longer period are nearly coincident in the two curves. Averages of the two series of epochs are given in table 3.

Brooks (8) made a periodogram analysis of the Nile flood data using the same list as that published in 1923, and found that a period of about 77 years is the only period that could be regarded as real, judging from the mathematical criterion.

The 300-year period.—Fifty-year means of the low stages of the Nile are shown plotted in figure 3. The 300-year variation is clearly evident. This long period can be seen also in the curve of flood stages and the maxima and minima of the two curves are virtually identical. The secular increase in the levels of both high and low stages is due to the raising of the Nile bed by the deposition of the silt which it brings down.

WHEAT PRICES IN ENGLAND

In a paper published after his "Klimaschwankungen", Brückner concluded from an examination of wheat prices in western Europe for 200 years that high prices occur during or shortly after periods of maximum rainfall. Beveridge (11) computed yearly index numbers of wheat prices in England from 1500 to 1870 by expressing them as a percentage of 35-year moving averages. His periodogram of wheat prices shows a period of considerable amplitude at 35.5 years.

I have taken Rogers' wheat prices in England from 1265 to 1700 and formed index numbers by expressing the 5-year means as a percentage of moving averages of 7 pentad means. From 1700 to 1870 5-year means of Beveridge's index numbers are employed. After 1870, the Sauerbeck index numbers are used. These pentad index numbers, smoothed by the formula $(2a+3b+2c) \div 7$, are shown graphically in figure 1. Table 2 gives the epochs of maxima and minima. These epochs are virtually identical with the epochs of wheat prices in my 1905 paper.

TREE GROWTH IN ARIZONA AND CALIFORNIA

Douglass was an early investigator of tree-growth in its relation to climate. Some of his early measurements were published in MONTHLY WEATHER REVIEW, June 1909. Huntington published in 1912 results of his measurements of the tree rings of the California Sequoias.

The 37-year period.—In his "Climatic Cycles and Tree-Growth," volume 1, Douglass gives a table of mean yearly growth of 5 yellow-pine trees measured near Flagstaff, Ariz., dated from 1503 to 1910 and of 2 trees from 1385 to 1503. This record appears to be quite homogeneous. Residuals of 5-year means of these measures from a smooth curve, formed by successive means of 8 values further smoothed by means of 2 terms, were smoothed by the formula $(a+2b+3c+2d+e) \div 9$ and the final values plotted in figure 1. The 37-year epochs derived from inspection of this curve are given in table 2. Other records from trees in New Mexico, Colorado, and Utah show this variation with epochs nearly synchronous with those of the Flagstaff region.

The 83-year period.—We are indebted to Huntington for an extensive series of measurements of the growth rate of the California Sequoias. His material has been worked over by Antevs (12) who divided it into two groups—"A", trees growing in dry situations; and "B",

trees growing in moist situations. His tables give the total growth for each decade from 1000 B.C.

The trees in moist situations seem to respond to changes in meteorological conditions affecting their growth sooner than those in dry situations, and their variations are somewhat more regular. For these reasons the "B" series of means are selected to show the 83-year period. The secular trend in these values has been eliminated by taking residuals from successive means of nine terms. These residuals smoothed by $(a+2b+c) \div 4$ are plotted in figure 2. The 83-year epochs selected from the original and smoothed curves are given in table 3. The epochs derived from the growth rate of trees in dry situations lag about 10 years after these epochs.

For the years previous to the Christian era, the data from trees in both dry and moist situations (Antevs' "C" group) were used, since the total number is small. The 83-year epochs are given in table 3.

The 300-year period.—To show this period, 50-year means of Huntington's Sequoia measurements are plotted in figure 3. The maximum at 800 is weak but is well marked in Antevs' curve "B".

CHINESE EARTHQUAKES

A number of catalogs of earthquakes are available for the study of their periodicities. The most extensive one is by Mallet in the British Association Report of 1858. Extensive catalogs of Chinese earthquakes have appeared but there are, especially in the later ones, too many entries for a single large shock, due to the aftershocks and to the large number of provinces reporting it, so that the list is unsuitable for analysis. Turner (9) made a periodogram analysis of the list of earthquakes in China compiled by Hirota, in British Association Report, 1908, since he regarded it as sufficiently homogeneous for this purpose. He pointed out that periods of around 79 and 284 years appeared probable. Hirota's list ends in 1645. Parker's list in British Association Report, 1909, extends from 1640 to 1875, but it lists only the greater shocks. It is, however, internally homogeneous and shows the 37-year period fairly well.

The 37-year period.—I have counted the number of shocks in these two lists for each 20-year period, as in the case of severe winters, and the number for each fifth year is plotted in figure 1. Between A.D. 195 and 225 there are no records owing to the Great Rebellion. Epochs of maxima are given in table 2.

The 83-year period.—Smoothing the earthquake numbers in the same manner as those of severe winters, the 37-year period is eliminated. These numbers by decades plotted in figure 2, show the 83-year period with epochs as given in table 3. Two epochs of maxima are interpolated. One at 220 occurred during the civil war and the other at 1250 is not obvious from the data which seem to be unusually scanty at this time. However, there is a pronounced maximum of Japanese earthquakes near this date. The maximum at 1630 is unreliable owing to the ending of the record around 1640 and evidence from other lists points to 1650 as a more probable date. The 83-year epochs after 1650 cannot be reliably determined.

The 300-year period.—The number of Chinese earthquakes per half-century from Hirota's list, and also the numbers derived from Mallet's list, are plotted in figure 3. Previous to 400 A.D., Mallet's data are too scanty to show the secular variation. The number of years per half-century with earthquakes in China, compiled from the list by Gauthier in Bull. de l'Observ. de Zikawei, 1907, is also plotted. This curve shows clearly the 300-year

variation and the other two curves are in fair agreement. The numbers in the first half of the third century were doubled owing to the hiatus in the records.

THE 300-YEAR PERIOD

The 300-year period has already been shown to exist in the variations of solar and certain terrestrial data. Other data from literary records have been brought together by Brooks and the variations in these data seem to fit in well with those of auroras, etc. In his "Climate through the Ages", table 22, under Europe (general) and Belgium, the percentage of a to $a + b$ is an index of the raininess of these regions. Similar indexes for the severity of winters in Belgium were computed by me from Vanderlinden's catalog. From Speerschneider's compilation the percentage of years with heavy ice in Danish waters and the number of winters with ice on the Danish coast were obtained. From Co Ching Chu (10) are derived indexes of the raininess and the number of severe winters in China since the first century A.D. These data are shown graphically in figure 3. To facilitate intercomparison of the variations in these curves, the 300-year maxima are indicated by circles and these are joined by broken lines.

While the data graphically shown in figure 3 are obviously of only limited accuracy there is sufficient agreement among the curves to show that *the epochs of cold, wet periods are around 200, 550, 850, 1125, 1350, 1625, 1850. The warm, dry epochs are approximately 350, 700, 975, 1250, 1500, 1725.* Brooks in his figure 38 gives a composite curve which he thinks represents the variations of rainfall over the Eur-Asian continent during historical times. The maxima of his curve are approximately 425 B.C., 125 B.C., 175, 525, 850, 1125, 1350, 1600, 1825. These dates agree well with the 300-year epochs derived from the curves.

The epochs of maximum acceleration of the 11-year epochs, derived from curve 4 and given in table 4, precede, 50 to 225 years, the epochs of maxima of rainfall. The lag is variable, being greatest around 800 and least around 1600.

THE 1400-YEAR PERIOD

A long period of approximately 1,400 years was noted above in the variations in the length of the 11-, 37-, 83-, and 300-year periods. The 83-year period in the aurora also gives clear evidence of this long period. Since the frequency of auroras correlates closely with that of severe winters in the shorter 37- and 83-year periods, there should be evidences of the long period in meteorological fluctuations. Brooks (*loc. cit.*) has brought together all available evidence relating to climatic fluctuations during the last 5,000 years. His results should furnish impartial evidence as to the existence of the long period, and a summary follows of the maxima and minima in his climatic curves which seem to recur at intervals averaging 1,400 years. His curves showing variations of rainfall in Europe and Asia indicate well-defined minima around 2200 B.C., 1000 B.C., and 600 A.D. Maxima are shown near 3000 B.C., 1300 B.C., between 800 B.C. and 350 B.C., and near 1300 A.D. The maximum in the first millenium B.C. is the so-called "Classical" rainfall maximum, and the maximum near 1300 is the "Medieval" rainfall maximum. According to Peake, as quoted by Brooks, a period of drought occurring some centuries before 3000 B.C. caused migrations from the interior toward the Baltic, while the great dispersal occurred about 2200 B.C. Brooks places the post-glacial "Climatic Optimum" at this time. Huntington's curve of tree-growth has chief maxima at 400 B.C. and

1300 A.D. and a minimum at 700 A.D. Brooks' curve of temperature in Europe shows a maximum about 700 A.D. and minima 0 to 250 B.C. and around 1500 A.D. The deterioration of climate in Greenland from about 900 A.D. to 1400 is consistent with these fluctuations in Eur-Asia and North America.

It is clear, therefore, that *marked climatic extremes have occurred in the Northern Hemisphere with intervals averaging 1,400 years.*

CORRELATION BETWEEN SOLAR AND TERRESTRIAL VARIATIONS

In figures 1 and 2, below the curves, the 37-year and 83-year epochs of minimum length of the 11-year solar period are plotted on their respective dates. Next below are the corresponding epochs of maximum frequency of the aurora. Then follow the epoch of maxima for severe winters, Nile levels, wheat prices, tree growth, and Chinese earthquakes. Connecting lines are drawn to show the relations and the varying lags. In general the lags are greater before 1000 A.D. than afterwards.

The 37-year period.—The lag of the auroral after the solar epochs averages 24 years—33 years before 1000 and 15 years after. The lag of the epochs of severe winters averages 56 years—70 before 1000 and 44 after. With reference to the epochs of severe winters, the lag of the Nile flood epochs is 1.5 years; that of wheat prices 5 years; that of Arizona pines 23 years. The relation of earthquakes to other terrestrial events is uncertain, but assuming that indicated by the lines, the maxima average 13 years earlier than those of severe winters.

The 83-year period.—The lag of the auroral after the solar epochs is 55 years; that of severe winters averages 91 years—107 before 1000, and 79 thereafter. The lag of the Nile stage after severe winters is 8 years, Sequoia growth 63 years. Chinese earthquakes precede severe winters by about 13 years. *It will be seen that the lags vary directly with the length of the period and that in the case of severe winters the lag after 1000 is about two thirds that previously.* A similar lag was noted above for the 300-year epochs of rainfall. This long-period variation in the lag is probably due to the 1,400-year period.

The 300-year variation in the lag.—There is a well-defined 300-year periodicity in the lag of the epochs of severe winters after the solar epochs in both 37-year and 83-year periods (cf. fig. 3). For the 37-year period, the maximum lag occurs around 360, 715, 975, 1335, 1700; minimum lag 580, 900, 1200, 1540, 1810. These epochs of maximum and minimum lag average 100 years after the epochs of short and long 37-year intervals in table 4. For the 83-year period, the maximum lag occurs around 400, 675, 860, 1235, 1610; minimum lag 525, 780, 1050, 1380, 1775; or about 90 years after the epochs of short and long 83-year intervals in table 4.

These consistent variations in the lags of the meteorological events and their persistency for 1,500 years afford additional proof of the reality of both the solar and meteorological periods.

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